



Texture analysis and classification using deterministic tourist walk

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ABSTRACT

In this paper, we present a study on a deterministic partially self-avoiding walk (tourist walk), which provides a novel method for texture feature extraction. The method is able to explore an image on all scales simultaneously. Experiments were conducted using different dynamics concerning the tourist walk. A new strategy, based on histograms, to extract information from its joint probability distribution is presented. The promising results are discussed and compared to the best-known methods for texture description reported in the literature.

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1. Introduction

Texture is an important visual attribute. It is used to describe images in computer vision and image processing. Applications using textures are found in various areas, ranging from aiding diagnoses in medical images [1], passing by remote sensing [2], analysis of geological structures in images [3], microscope images [4], etc. Texture is a visual pattern attribute. It consists of sub-patterns, which are related to the pixel distribution in a region and characteristics of the image object, such as size, brightness and color. Even though, there is no exact definition for the term texture, this is an attribute easily comprehended by humans and responsible for extracting meaningful information from images. The importance of texture perception is presented in Ref. [5] from the viewpoint of human vision and practical computer vision applications.

Many methods have been proposed in the literature for an efficient texture description [5,6]. These methods are, in general, based on the spectral analysis of the pixels of the image (e.g., Fourier descriptors [7] and Gabor filters [8]), statistical analysis of the pixels (e.g., co-occurrence matrices [9], Local Binary Pattern [10], Feature-based Interaction Map [11]) and complexity analysis (e.g., Fractal Dimension [12–14]). However, most of the proposed methods focus on micro-texture analysis (analysis of a small set of pixels).

The number of methods applied to macro-textures is reduced, due to the inherent difficulty in the analysis [3].

Recently, we have proposed a novel method of texture analysis able to explore different scales in the image, the deterministic tourist walk [15,16]. It considers independent walkers leaving from each pixel of an image. For a given memory, a walker moves to one of its neighboring pixels according to the difference of intensity between these pixels. Each generated trajectory, after a transient time, ends in an attractor, i.e., a cycle of pixels from where the walker cannot escape. These attractors contain characteristics of the pixel organization in that image region. Here we present a novel approach for texture analysis and classification based on the analysis of the transient time and cycle period joint probability distribution computed by the deterministic tourist walk.

We start presenting an overview of the deterministic tourist walk in Section 2. In Section 3, the method is described in detail for image applications as well as the problems of detecting an attractor during a walk. A novel attractor detection methodology is proposed. In Section 4, a study of the dynamics of the tourist walk over texture images is presented. We also show how to build texture signature vectors from the joint transient time and cycle period probability distributions. Experiments using synthetic and natural texture images are proposed in Section 5 and results are presented in Section 6. Finally, in Section 7, the conclusions and improvement of the method are discussed.

2. Deterministic tourist walk (DTW)

Random walks over regular lattices and random media have often been studied and have a wide range of applications [17,18].

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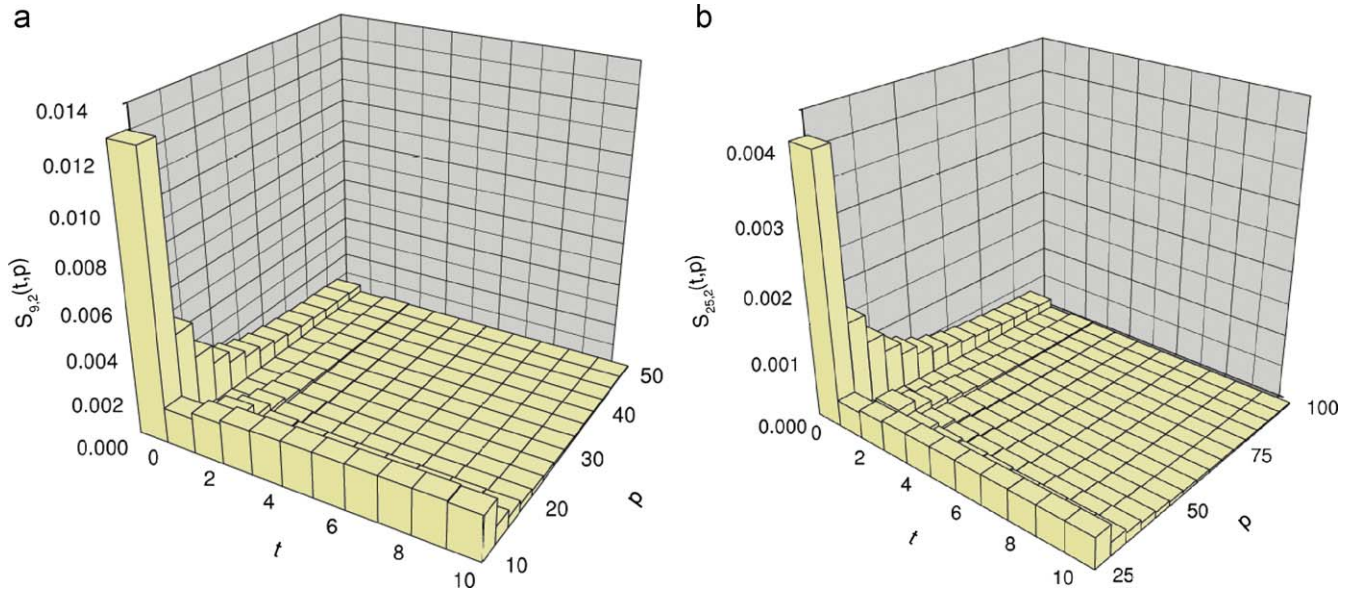


Fig. 1. Joint probability distribution of transient time (t) and period (p) for two values of μ with (a) $\mu = 9$ and (b) with $\mu = 25$. One observes that the general aspects of both histograms are similar.

Although it is not so thoroughly investigated, deterministic walks in regular [19,20] and disordered media [21] also present very interesting results. Here, we are interested in a partially self-avoiding deterministic walk algorithm, which we call the deterministic tourist walk (DTW) [22,23].

The tourist walk algorithm can be understood as a walker (tourist) wishing to visit N points distributed in a map of d dimension. These points can be considered as sites and a tourist can move on them. The tourist follows the deterministic rule of, at each discrete time step, going to the nearest site not visited in the previous μ steps, a walker performs a partially self-avoiding walk, where the self-avoidance is limited to the memory window $\tau = \mu - 1$. This quantity represents a characteristic time to the site to become attractive to the walker again (refractory time) and prohibits a trajectory from intersecting itself inside this memory window.

The tourist's behavior depends strictly on the data set configuration and on the starting site. The tourist's movements are entirely performed based on a neighborhood table. This table represents the *tourist graph*, i.e., nodes with τ fixed directed and weighted outgoing links (edges) each and with a variable number of incoming links (edges). Notice that the distances among the sites are simply a way of ranking their neighbors. This feature leads to an invariance in scale transformations [24].

Each tourist walk has an initial transient part of length t and ends in a cycle with period p . Both the transient time and cycle period can be combined in the joint probability distribution $S_{\mu,d}^{(N)}(t,p)$. Fig. 1 shows the joint probability distribution for $\mu = 9$ and 25.

Next, we consider some special cases obtained combining different values of μ and d .

The simplest case to deal with the DTW is to consider $\mu = 0$. This case is trivial, since the walker has a null-size memory. The walker remains at the same site. The trajectory has a zero-length transient and a cycle of period $p = 1$. The transient and period joint probability distribution is simply given by

$$S_{0,d}^{(N)}(t,p) = \delta_{t,0} \delta_{p,1}, \quad (1)$$

where $\delta_{i,j}$ is the Kronecker's delta. Despite its triviality, this becomes interesting because it is the simplest situation of the stochastic tourist walk [25].

For a memoryless tourist ($\mu = 1$), the walker only knows the nearest neighbor of its current site. Thus, at each time step, the walker must leave the current site and go to the nearest one. The name "memoryless tourist" is devised because the walker knows the site where the walker is, but does not remember any of the previously visited ones. This rule does not lead to exploration of the random medium, since after a very short transient time, the walker becomes trapped by a couple of mutually nearest neighbors. The transient time and period joint probability distribution was analytically obtained for $N \gg 1$ [26]:

$$S_{1,d}^{(\infty)}(t,p) = \frac{\Gamma(1 + I_d^{-1})(t + I_d^{-1})}{\Gamma(t + p + I_d^{-1})} \delta_{p,2}, \quad (2)$$

where $\Gamma(z)$ is the gamma function and $I_d = I_{1/4}[\frac{1}{2}, (d+1)/2]$ is the normalized incomplete beta function. Analytical calculations [27] were also performed for the stochastic tourist walk.

An interesting phenomenon occurs when greater values of μ are considered. In this case, the cycle distribution is no longer peaked at $p_{\min} = \mu + 1$, but presents a whole spectrum of cycles with period $p \geq p_{\min}$, with possible power-law decay [22,23,28–30].

3. Deterministic tourist walk on images

Consider a digital image of $M_x \times M_y$ size and $N = M_x \times M_y$ pixels, where each pixel (x,y) is associated to a gray level ranging from 0 to 255. Two pixels, (x_i, y_i) and (x_j, y_j) are considered neighbors if the geometrical distance between them is smaller than 2, i.e., $d(i,j) < 2$, where $d(i,j)$ is the Euclidean distance:

$$d(i,j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \quad (3)$$

Once two pixels are considered geometric neighbors, the module of the difference of their intensities is defined as the real "distance" between them [15,16].

Now consider a traveler walking through neighboring pixels. This traveler can only walk according to the following rule: move to the nearest or furthest neighbor (i.e., the one which differs in the minimum or maximum value, respectively, from the current position) and that has not been visited in the last μ ($\mu \in [1, N]$) previous steps.

This rule produces partially self-avoiding walks called deterministic tourist walks (Section 2).

Trajectories yielded from these walks are divided into two parts: an initial part, with t steps, called transient, and the final part, where the traveler is trapped in a cycle of period $p \geq \mu + 1$, called attractor. In images, these attractors consist of a group of pixels which compose a path from where the tourist cannot escape. Although, there are cases where, depending on the disposition of pixels along the image and the memory size μ used, the tourist cannot find an attractor. In this case, the tourist walks until it finds a transient with a size equal to the number of image pixels ($t = N$) and the resulting trajectory is considered as having only the transient part ($p = 0$).

Another common situation found in images is the existence of two or more directions complying with the tourist walking rule. In this case, the draw is solved by selecting the first direction, among the drawn directions, when the neighbors are visited in a clockwise order according to the scheme presented in Fig. 2. This approach preserves the deterministic characteristic of the method.

For each initial condition (i.e., the starting pixel), the tourist produces a different trajectory. Notice, however, that different initial conditions can lead to the same attractor. Considering all pixels in the image as starting points, we compute the joint probability distribution of transient time t and attractor period p , $S_{\mu,2}^{(N)}(t,p)$ (Fig. 3).

8	1	2
7	1	3
6	5	4

Fig. 2. Visiting order of neighbors for a tourist placed at pixel i .

From the study of these distributions using statistical techniques it is possible to achieve a signature able to discriminate the image texture [15,16,24].

3.1. Attractor detection

One of the most challenging tasks in the study of the deterministic tourist walk is the attractor detection. An attractor is a cycle of period p that exists at the end of a tourist trajectory. It is a walk section that starts and ends at the same image pixel and from where the tourist is unable to escape [22,24,30,30].

Considering the attractor as a walk section that starts and ends at the same image pixel may lead one to think that, once it visited a pixel, a new visit would configure an attractor. Nevertheless, this is a very simple, and likely to fail, approach for attractors' detection. In fact, during a walk an image pixel can be re-visited without configuring an attractor. Besides, the tourist finite memory μ , which indicates the pixels visited in the last μ steps and that cannot be visited at the present time, allows some steps of the walk to be repeated without configuring an attractor. This characteristic enables sophisticated tourist walks, but it also increases the difficulty of detecting an attractor, and thus requiring more accurate methodologies (Fig. 4).

Repeating walk steps, without configuring an attractor, requires that different parts of a walk be compared in order to identify the presence of a repeated section. This repeated section characterizes an attractor. This comparison is made by keeping all pixels visited at this point of the tourist walk in list L . This list enables us to check the existence of repeated sections and, as a consequence, the presence of an attractor in the walk (Fig. 5).

However, comparing sections of this list L is a time consuming task. At each new step, it is necessary to check if there is a sequence of steps $p \geq \mu + 1$ repeated throughout the list L . An alternative to this exhaustive search, or “brute force” method, is, at first, to check if the current step of the tourist has been visited at least three times and,

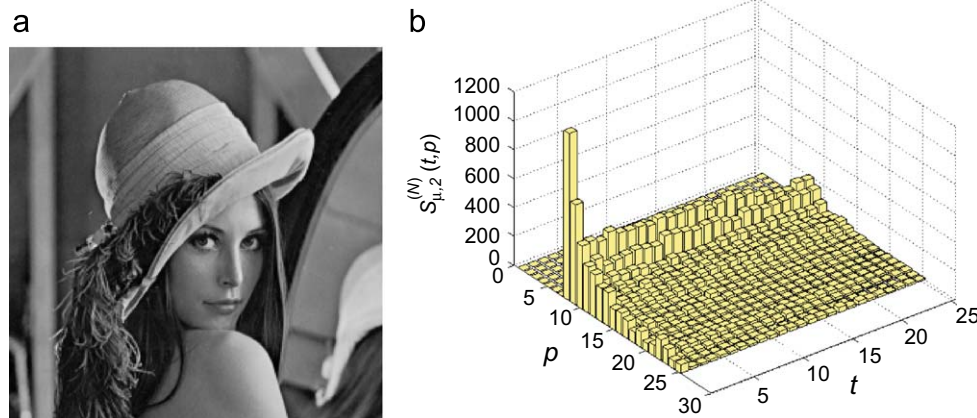


Fig. 3. (a) Original image; (b) Joint probability distribution of transient time and attractor period for $\mu = 5$.

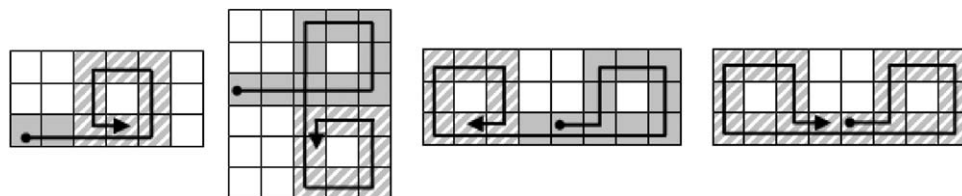


Fig. 4. Examples of possible tourist walks. Gray part represents the transient part while the striped pixel represents the attractor part.

in an affirmative case, to check the sections restricted by this step. An attractor is defined by a sequence of steps repeated in the walk and this sequence starts and ends at the same position, so at least three repetitions of the initial step (x,y) are necessary to characterize two repetitions of an attractor in the walk (Fig. 6). This existence condition enables us to optimize the search in list L and it makes it faster and easier to detect the attractor.

In order to check the repetition of a step in the walk, a new list is associated to each step. This new list is responsible for keeping

1	2	3	4	5	6	7	8	9	10	11	12
P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₄	P ₅	P ₆	P ₇	P ₄

L

Fig. 5. List L used to keep track of the walk visited sites.

1° Repetition							2° Repetition				
P ₁	P ₂	P ₃	P ₄	P ₅	P ₆	P ₇	P ₄	P ₅	P ₆	P ₇	P ₄

L

Fig. 6. Comparison between different parts of list L . A step (gray) repeated at least three times in this list configures the existence of repeated sequences and, probably, an attractor.

P ₁	1
P ₂	2
P ₃	3
P ₄	4 8 12
P ₅	5 9
P ₆	6 10
P ₇	7 11

Fig. 7. List associated to each step from the walk kept in L .

the positions where the (x,y) shows up in the list L (Fig. 7). This enables us to check the existence of repeated steps quickly as well as the number of repetitions, therefore, reducing the computational cost involved in the attractor detection task.

3.2. Attractors on images

To investigate the behavior of a tourist during its walk, an experiment was performed. This experiment consists of examining the attractors generated, for different memories μ , when the tourist walks on different environments: a random generated image and a texture image. Both used images have 100×100 size and 256 gray levels.

First, we realize that the memory has a great influence on attractor distribution in both images. For small μ values, we note a higher number of attractors distributed on the images. These attractors present a simpler behavior and small repetitions of sections. Otherwise, as the memory increases, the number of attractors decreases, i.e., a greater number of walks tend to find the same attractor. As the memory increases further, the tourist is forced to look for bigger attractors. These new attractors are present in a smaller proportion in the image, and they also present a more complex behavior (Figs. 8–11). We also note that the tourist does not require a long walk to find an attractor. This is shown by the number of attractors that present a transient with $t = 0$ (Fig. 12).

On random generated images, a tourist walk is characterized by chaotic behavior. This behavior is due to the environment where the tourist is, i.e., as there is no correlation between neighboring pixels, the tourist walks at random and changes its direction with the same frequency as the environment changes (Figs. 8 and 9). However, digital images present a visual context, i.e., pixels are correlated so that a scene, the face of a person or a texture pattern, is composed. This correlation between neighboring pixels influences the choices of directions taken by the tourist during the walk.

An important issue about the tourist walks is the walking rule adopted by the tourist. When a tourist starts its walk on an image, it must choose to go to the neighboring pixel which is the nearest

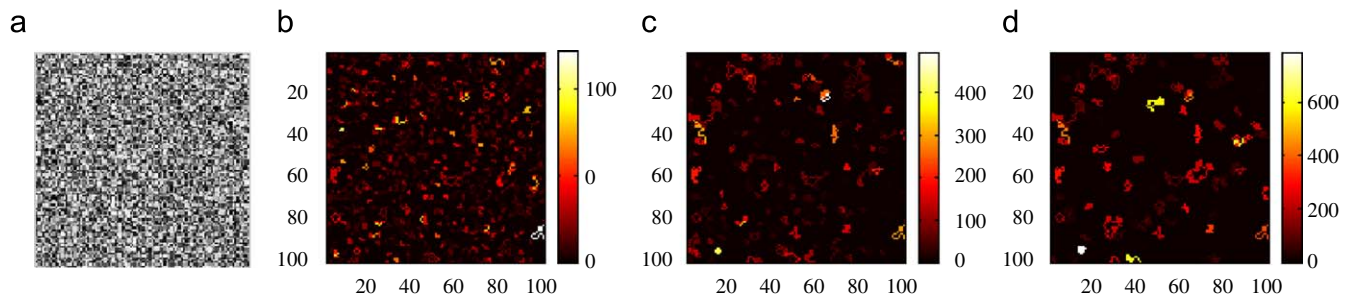


Fig. 8. Attractor generated by the tourist on a random generated image using the minimum distance and different μ values: (a) Original image; (b) $\mu=1$; (c) $\mu=3$ and (d) $\mu=7$.

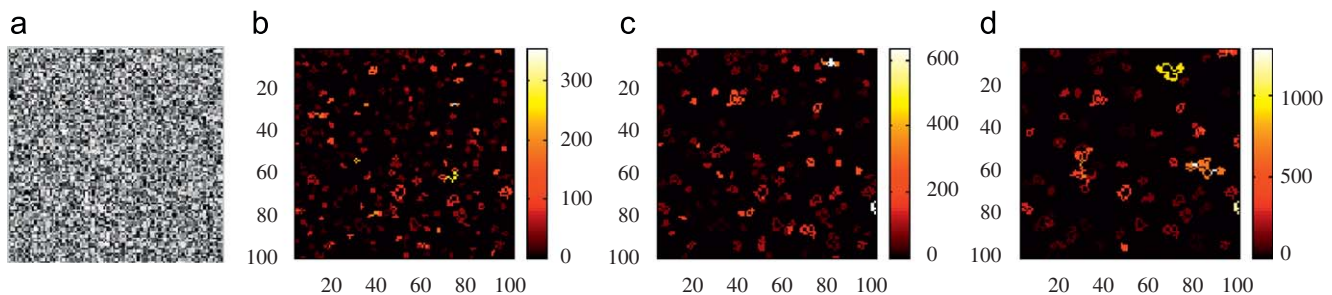


Fig. 9. Attractor generated by the tourist on a random generated image using the maximum distance and different μ values: (a) Original image; (b) $\mu=1$; (c) $\mu=3$ and (d) $\mu=7$.

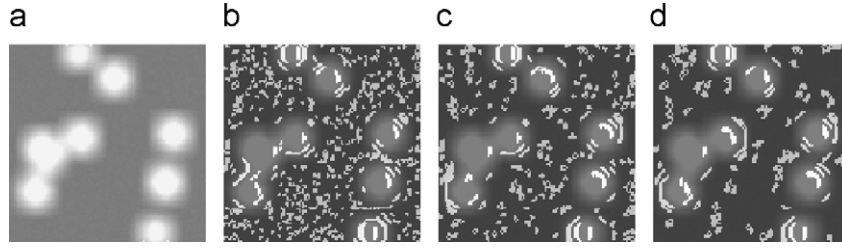


Fig. 10. Attractor generated by the tourist on a texture image using the minimum distance and different μ values. Attractors are mixed with texture for better comprehension: (a) Original image; (b) $\mu = 1$; (c) $\mu = 3$ and (d) $\mu = 7$.

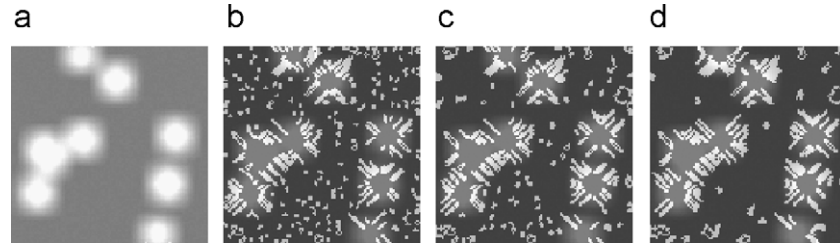


Fig. 11. Attractor generated by the tourist on a texture image using the maximum distance and different μ values. Attractors are mixed with texture for better comprehension: (a) Original image; (b) $\mu = 1$; (c) $\mu = 3$ and (d) $\mu = 7$.

or furthest (i.e., the pixel which presents the minimum or maximum difference of intensity, respectively, with respect to the current tourist position). Besides, once its rule is defined it cannot be changed. As a result, we note that walks guided to directions of minimum and maximum differences generate distinct attractor patterns for the same image.

The walks guided to the minimum difference of intensity are inclined to locate attractors in regions of the image which present higher homogeneity, therefore, avoiding regions with high contrast, such as an edge, and texture pattern changes (Fig. 10). Otherwise, the walks guided to the maximum difference emphasize attractors located in regions of lower homogeneity, i.e., heterogeneous regions and abrupt changes in image context (e.g., changes in texture or illumination of a region, or presence of edges) (Fig. 11).

This behavior yielded from the walking rules adopted reflects in the resulting joint probability distribution. This enables us to use the information stored in the joint probability distribution as a feasible signature for texture characterization and classification.

4. Texture analysis with DTW

Our preliminary results have shown that the transient time and cycle period joint probability distribution demonstrates a potential use in texture classification [15,16]. Fig. 13 shows the t and p joint probability distribution of three different texture patterns. One can visually notice the discrimination of the texture classes by the form of the surfaces. Nevertheless, a more quantitative index is difficult to extract from the surfaces [15,16]. Indeed, we have succeeded, but the results we present here are substantially better.

Here, we propose to use the histogram $h_\mu(n)$, where $n = t + p$. This histogram is computed from the joint probability distribution achieved for a specific μ value, and it represents the number of tourist walks, which have a size equal to $(t + p)$ in the joint probability distribution, where t and p represent the transient time and attractor period, respectively. Notice that $n = t + p$ is the number of visited pixels, but it does not exactly match the number of different pixels visited since the trajectories can cross themselves. According to the texture pattern present in the image and the memory μ used, a new

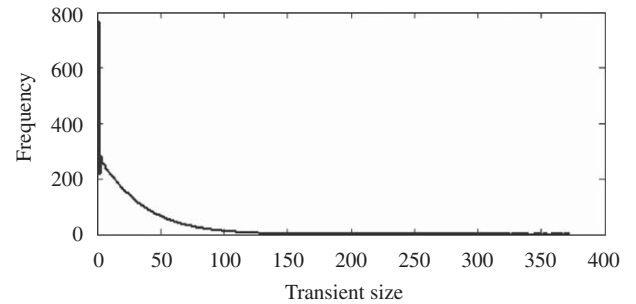


Fig. 12. Histogram of the transient size for $\mu = 4$.

joint probability distribution is achieved, as a consequence and a different histogram is computed, which makes it a useful tool for texture analysis (Fig. 14).

There is a relation between the histogram and texture behavior. In textures with well defined and constant patterns in the image, near attractors are favored. In this situation, the histogram presents a higher peak in the beginning of the curve that decays rapidly and in the end, the curve presents low values, as there are few long walks (see bottom texture pattern in Fig. 14). On the other hand, textures with sparse and not constant patterns, the probability of a walker to find an attractor varies according to the region of texture, making a more uniform histogram (see the middle texture pattern in Fig. 14). Furthermore, it can be observed that the initial few values are already sufficient to characterize the textures.

To use the tourist walk as a feasible texture signature, a feature vector $\vec{\psi}$ is constructed from the joint probability distribution for a specific μ value. A total of m descriptors are selected from walking histograms to compose the feature vector $\vec{\psi}$. As there are no attractors of size smaller than $(\mu + 1)$, the first descriptor selected has a size $(\mu + 1)$:

$$\vec{\psi}_\mu(n) = [h_\mu(\mu + 1), h_\mu(\mu + 2), \dots, h_\mu(t + p), \dots, h_\mu(\mu + m)].$$

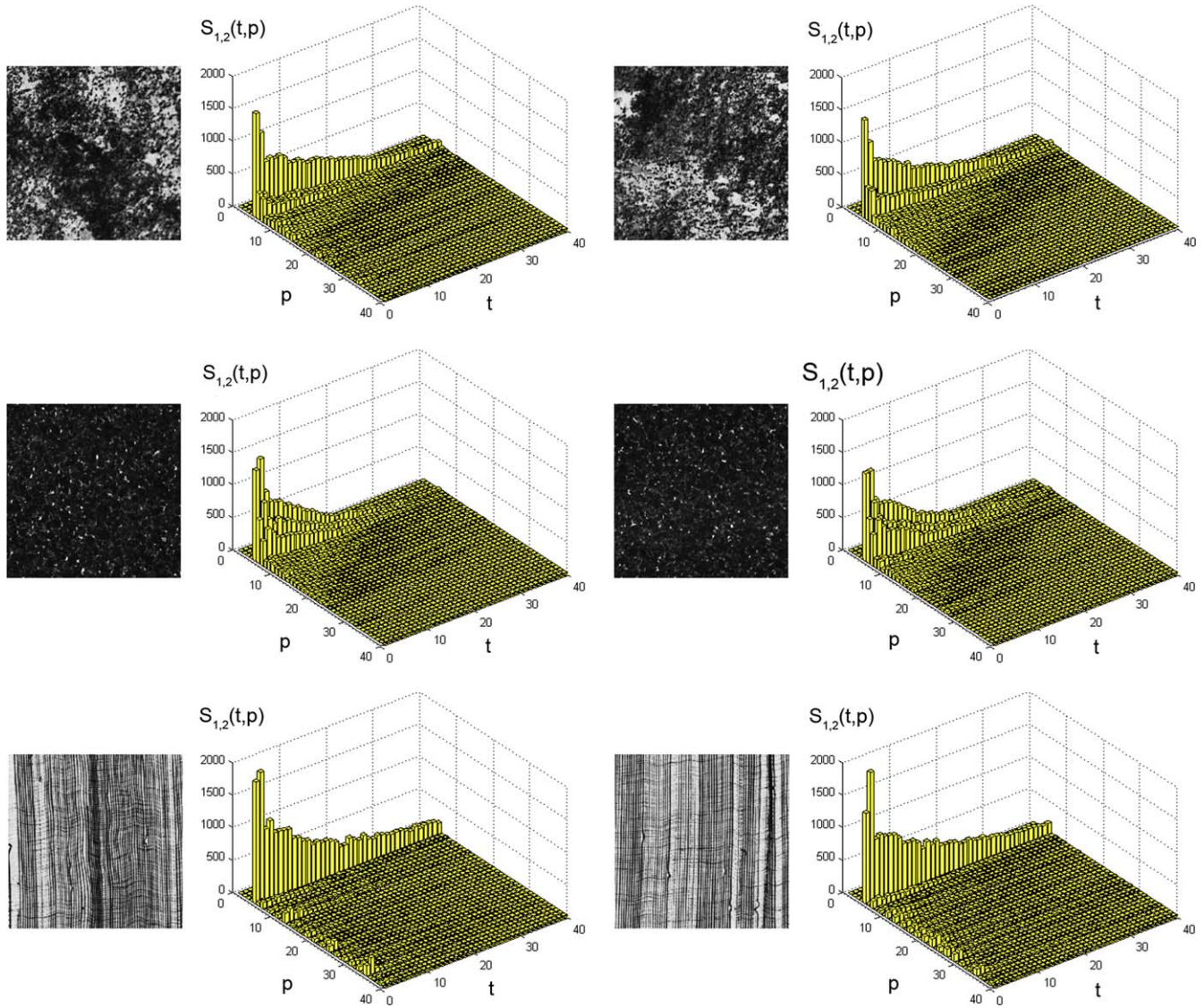


Fig. 13. Example of t and p joint probability distribution computed for three different texture classes using $\mu = 1$.

The joint probability distribution depends on the μ value, so we also propose an image signature considering different μ values. This texture signature consists of a concatenation of the signatures calculated using $\psi_{\mu}(m)$, for different μ values:

$$\vec{\varphi}_{\mu_1, \dots, \mu_M}(m) = [\psi_{\mu_1}(m), \psi_{\mu_2}(m), \dots, \psi_{\mu_M}(m)].$$

Statistical analysis has shown that joint probability distribution concentrates most image information on a few elements. These elements are located in a region where $0 \leq t \leq 4$ and $(\mu + 1) \leq p \leq (\mu + 4)$. Therefore, a total of $m = 4$ histogram descriptors were considered to compose the feature vectors ψ . Additional information about the influence of the memory on the transient time and cycle period distribution and its use in sample classifications can be found in Ref. [15].

5. Experiments

Signatures were analyzed using Linear Discriminant Analysis (LDA) in a leave-one-out cross-validation scheme. The LDA method can estimate a linear subspace where the projection of the data

presents larger variance inter-classes than the variance intra-classes. Additional information can be found in Refs. [31,32]. The leave-one-out cross-validation scheme separates one sample from a given class while supervision training and validation tasks are performed on the complementary sample. Once the training is completed, the remained sample is tested. Repeating this procedure, a success index can be obtained.

Tourist signatures were evaluated using two image databases: (i) synthetic textures and (ii) natural textures. The first database was made using images from the book of Brodatz [33], a set of images broadly used in computer vision and image processing literature as benchmark for texture analysis. In this experiment, each image has 200×200 pixels with 256 gray levels. A total of 40 classes, with 10 samples each, were used. One example of each texture class is shown in Fig. 15.

The second database used textures extracted from plant leaves of five different species from Brazilian flora. From each species, 10 texture samples of 128×128 pixels with 256 gray levels were extracted for the experiment, making a total of 50 images in the database. Examples of each texture class are shown in Fig. 16. The main motivation of this experiment was to evaluate the algorithm in a real

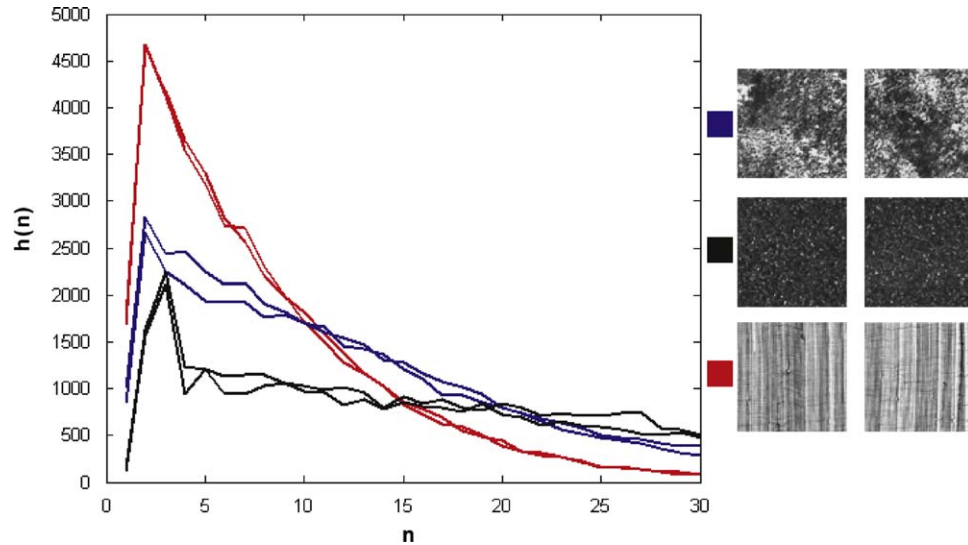


Fig. 14. Example of histograms computed for three different texture classes using $\mu = 1$.

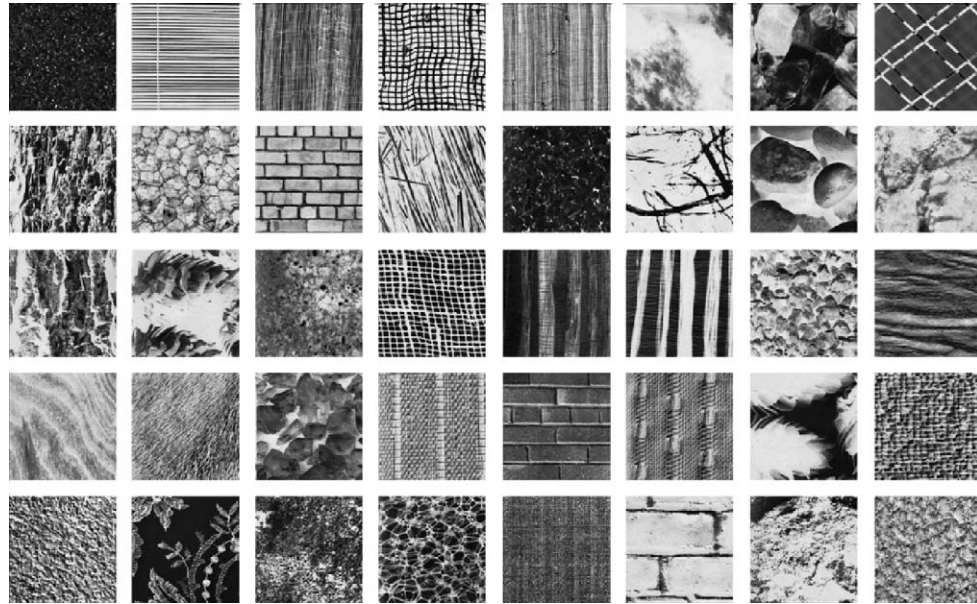


Fig. 15. Example of 40 Brodatz texture classes used in the experiment. Each image has 200×200 pixels and 256 gray levels.

and complex problem. Developing computer tools that can identify plants is a current scientific challenge. Identifying plant leaves, as well as, the texture of leaves is a very difficult task, considering the high variability inside the same class and the similarity between classes [34,35].

To evaluate the proposed method better, a comparison with traditional texture analysis methods was made. The following methods were considered:

Fourier descriptors: In the experiment, these descriptors consist of a feature vector providing the energy of the 99 most meaningful coefficients of the Fourier Transform. Each coefficient corresponds to the sum of the spectrum absolute values placed at a radial distance from the center (after performing a *shifting* operation) of the bi-dimensional transformation [7].

Co-occurrence matrices: These matrices represent the joint probability distributions between pairs of pixels at a given distance and direction. In the experiments, we considered distances of 1 and 2

pixels with angles of -45° , 0° , 45° , 90° , in a non-symmetric version. Energy and entropy were computed from resulting matrices totalizing a set of 16 descriptors [9].

Gabor filters: This approach convolves an image by a family of Gabor filters (i.e., different scales and orientations from the same original configuration). Each Gabor filter is defined as a bi-dimensional Gaussian function modulated with an oriented sinusoid in a determined frequency and direction. For the experiments, a family of 16 filters (four rotation filters and four scale filters), with frequency ranging from 0.01 to 0.30, were used. Additional information can be found in Refs. [8,36–38].

6. Results

Next, we show that a combined minimum and maximum signature lead to better image retrieval. We start considering synthetic and then natural textures.

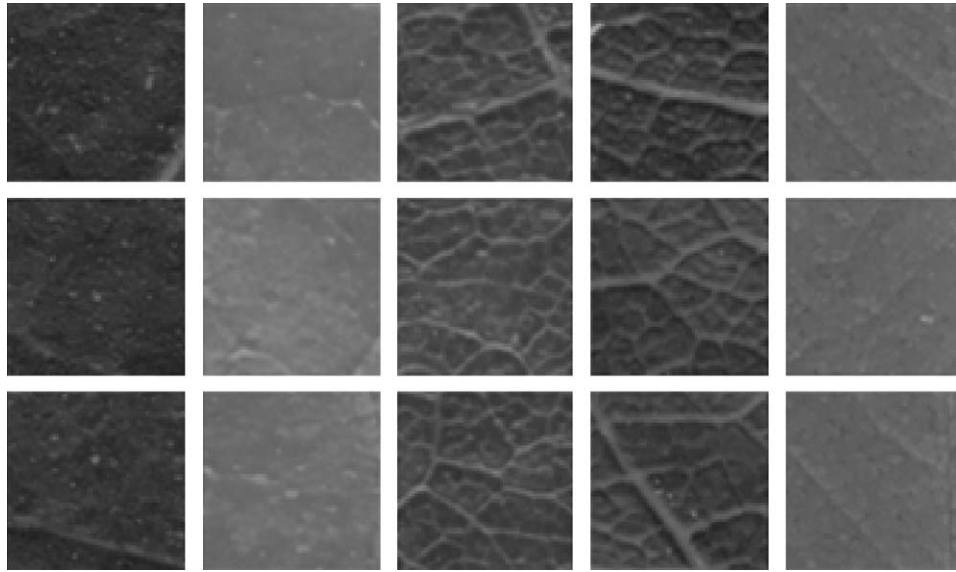


Fig. 16. Example of plant leaf textures used in the experiment. Each column represents a different texture class, while rows represent the class variation.

Table 1

Success rate for $\tilde{\psi}_\mu$ signature using different μ values and walking rules in the Brodatz database.

	Memory (μ)					
	0	1	2	3	4	5
Min	58.25	47.50	44.75	41.00	32.75	34.25
Max	78.50	67.50	64.25	56.00	54.00	45.25
Min $\cup$ Max	87.50	86.50	85.00	76.75	69.25	63.25

6.1. Synthetic textures

Table 1 shows the results of the tourist walk on the Brodatz texture image database for different μ values. We note that tourist walk presents a better result when it is conducted in the direction of the maximum difference of pixel intensity, instead of the minimum difference. In this case, tourist attractors are formed in heterogeneous regions, i.e., regions where the pixel intensity changes in an abrupt way, which characterizes the presence of image contours or changes in texture patterns. In fact, contour is one of the most important visual attributes to characterize objects, as it provides the most relevant information of an object for both identification and classification tasks [39,40]. However, combining minimum and maximum signatures into one (Min $\cup$ Max) leads to an increase in the success rate. This new signature presents both heterogeneous and homogeneous image information, which provides a more powerful tool for image analysis.

Results also show that, for both maximum and minimum difference and their combination, the success rate decreases as the memory μ increases. Explanation for this result lies in the fact that, as the memory increases, the more difficult it is to find an attractor in the image. Now the tourist is compelled to walk more to find an attractor and there is an increase in the attractor period (size). Besides, long walks may lead to a trap. In this case, the tourist does not find an attractor, which changes its joint probability distribution and affects the classification of the samples. Small memory values can have a better local analysis of the image texture which reflects in the higher success rate yielded.

Table 2 shows results yielded when multiple μ values are considered. Therefore, signatures calculated using $\tilde{\psi}_\mu$ for different μ values, are concatenated to compose a φ signature. This approach

Table 2

Success rate for the φ signature combining different μ values in the Brodatz database.

Memories used (μ)	Min	Max	Min $\cup$ Max
{0, 1}	73.25	87.00	92.75
{0, 1, 2}	81.50	89.50	95.00
{0, 1, 2, 3}	82.50	90.00	94.75
{0, 1, 2, 3, 4}	83.00	90.00	95.25
{0, 1, 2, 3, 4, 5}	84.00	90.50	95.50

Table 3

Comparison results for different texture methods in the Brodatz database.

Method	Images correctly classified	Success rate (%)
Fourier	351	87.75
Co-occurrence matrices	330	82.50
Gabor filters	381	95.25
Tourist walk	382	95.50

diminishes the individual importance for each μ value, but stresses the role of small values either for μ or the transient time and cycle period, thus providing a more efficient image classification.

Results yielded for each method compared are presented in Table 3. For this comparison, we consider the configuration that leads the tourist walks to the best results (φ signature using $\mu = \{0, 1, 2, 3, 4, 5\}$ for minimum and maximum walk rules). We note that the tourist's best result overcomes traditional methods (such as Fourier descriptors and co-occurrence matrices), while it presents a similar performance to the Gabor filters. This shows the great potential for image analysis and classification applications of the method.

Table 4Success rate for $\tilde{\psi}_\mu$ signature using different μ values and walking rules in the plant leaf database.

	Memory (μ)					
	0	1	2	3	4	5
Min	70.00	64.00	56.00	48.00	52.00	38.00
Max	68.00	66.00	68.00	64.00	70.00	52.00
Min $\cup$ Max	80.00	74.00	76.00	80.00	74.00	48.00

Table 5Success rate for the φ signature combining different μ values in the plant leaf database.

Memories used (μ)	Min	Max	Min $\cup$ Max
{0, 1}	74.00	86.00	92.00
{0, 1, 2}	68.00	82.00	96.00
{0, 1, 2, 3}	68.00	86.00	96.00
{0, 1, 2, 3, 4}	78.00	86.00	98.00
{0, 1, 2, 3, 4, 5}	78.00	88.00	92.00

Table 6

Comparison results for different texture methods in the plant leaf database.

Method	Images correctly classified	Success rate (%)
Fourier	30	60.00
Co-occurrence matrices	44	88.00
Gabor filters	38	76.00
Tourist walk	49	98.00

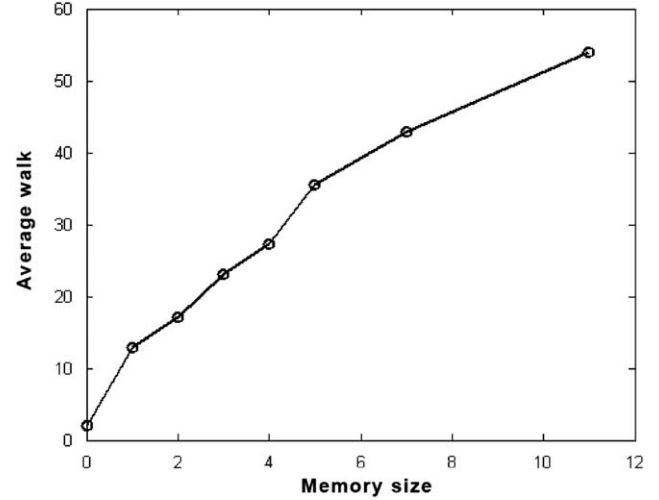
6.2. Results for natural textures

Table 4 shows the results of the tourist walk over the Plant Leaves texture database for different μ values. Results for this database corroborate the results from the previous experiment concerning the dynamics of the tourist and attractors: a better classification is yielded when the tourist is conducted to the direction of the maximum difference of pixel intensity, instead of the minimum difference. As in the previous experiment, this experiment confirms that combining minimum and maximum signatures into one (Min $\cup$ Max), it improves the discrimination power of the method, and it allows a better classification of the samples.

Table 5 shows results when signatures consisting of multiple μ values, $\tilde{\psi}_\mu$, are used. Results confirm this approach as a good strategy for the tourist method, once it provides a substantial increase in the success rate and image classification for both synthetic and natural texture patterns.

Comparison results are presented in Table 6. For this comparison, we consider the configuration that leads the tourist walks to the best results (φ signature using $\mu = \{0, 1, 2, 3, 4\}$ for minimum and maximum walk rules). As in the synthetic texture results, the tourist's best result overcomes traditional methods, including Gabor filters. It is important to emphasize that the leaf texture classification is a difficult task due to the small variation between classes (Fig. 16) and high variation inside each class [35]. This shows that the tourist walk is more efficient to deal with similar texture patterns than other textures methods compared by stressing small variations in the texture.

On one hand, macro-texture is the main feature to discriminate textures among the synthetic images. On the other hand, in the natural leaf texture images, the micro-texture is the main feature. The best results for synthetic images were achieved by the tourist walk and Gabor methods. The nature of the Gabor filter is based on the Gaussian function, which intrinsically blur an image. For macro-textures, this blurring does not corrupt the texture information. Nevertheless, in micro-textures this presents a serious objection. The tourist walk method deals with image information without filter-

**Fig. 17.** Average walk length for different memory μ values.

ing it by underlying function. Therefore, macro- and micro-texture information is always preserved.

6.3. Computational complexity

Tourist walks are performed for each pixel from an image. Considering an image of $N \times N$ size, this leads to N^2 walks. Each tourist walk consists of a transient part, of size t , and, if it exists, an attractor of size $p \geq \mu + 1$. In the case where an attractor cannot be found, the tourist walks until it finds a transient with a size equal to the number of image pixels, i.e., $t = N \times N$ and $p = 0$. Computational complexity of the tourist walk is determined by the number of image pixels and size of each walk, $O(N^2(t + p))$. The best case of the algorithm is achieved when all walks start on an attractor ($t = 0$) and the attractor presents a minimum size. The attractor size depends on image context and memory μ . For $\mu = 0$ and considering an opportune image context, the attractor size is minimized to $p = 1$, which leads to complexity $O(N^2)$ in the best case. The worst case is achieved when no attractor is found during a walk. In this case, for any memory size μ , the tourist walk presents size $t + p = N^2$, which leads to complexity $O(N^4)$. It is important to emphasize that the worst case is a very rare case, which requires a very specific configuration of pixels in the image. Even a random generated image does not produce this special case of walk.

Fig. 17 shows the average walk length for different memory μ values. For a memory value $\mu = 11$, the average walk length is equal to 55 steps. This result leads to a complexity which is very close to the tourist's best case, $O(N^2)$, and it is an excellent result in comparison to the complexities of Gabor filters ($O(N^2 \log N)$, due to the Fourier Transform) and co-occurrence matrices ($O(N^2)$).

7. Conclusion

This paper presented a novel approach of texture feature extraction based on the deterministic tourist walk. This method uses a

traveler to explore an image on a given scale (memory), where the tourist trajectory depends on the walking rule and the image context. Based on the studies of walk dynamics, a signature is computed from its resulting joint probability distribution. The yielded signature was tested in experiments using linear discriminant analysis to classify a set of synthetic (Brodatz) and natural (plant leaf) textures. Results show a great potential of the method as a feasible texture analysis methodology.

A color image consists of various matrices, which represent a chromatic scheme. The DTW can be applied to each one of these matrices and a combined signature vector can be made. Thus, the presented methodology can be generalized for color images.

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